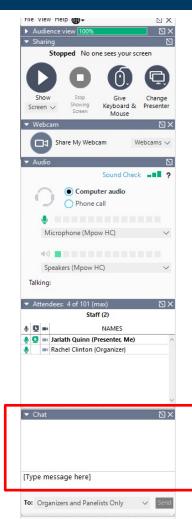


## How to interpret 'significance tests'

Jarlath Quinn – Analytics Consultant

## FAQ's

- Is this session being recorded? Yes
- Can I get a copy of the slides? Yes, we'll email a PDF copy to you after the session has ended.
- Can we arrange a re-run for colleagues? Yes, just ask us.
- How can I ask questions? All lines are muted so please use the chat facility – if we run out of time we will follow up with you.













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  - Gaming
  - Utilities
  - Insurance
  - Telecommunications
  - Media
  - FMCG



## **Agenda**

- What do we mean by 'Significance Testing'?
- Inferential Statistics and Significance Testing
- How to interpret P values
- How does a Chi Square test actually work?
- Correlations When is 'significant' not that significant?
- Interpreting confidence intervals correctly



## 'Significance' is a troublesome term

## Sir Ronald Fisher

- Introduced the phrase "...tests of significance"\*
- Unfortunately, saying that something is 'statistically significant' is likely to be misunderstood
- In this context, 'significance' does not mean notable
- Sometimes it doesn't even mean it's reliable
- Even the term 'test' is slightly problematic
- Significance Tests are more properly described as probability of evidence estimates

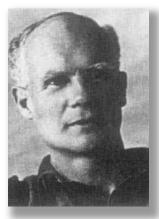




## The Usual Suspects









R.A. Fisher

Karl Pearson

**Egon Pearson** 

Jerzy Neyman

Alpha level **Confidence Intervals** 

Principal component analysis Reject / Accept

0.05

Maximum

num Exact Tests Significance

Likelihood Null Hypothesis

Chi Square

Design of

F Test

Stratified sampling

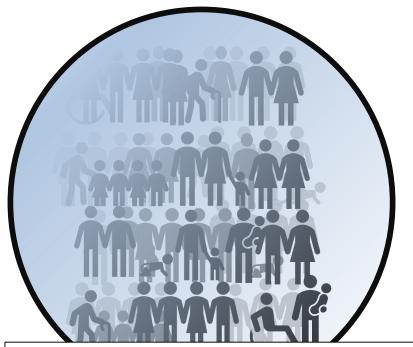
Z tests

experiments



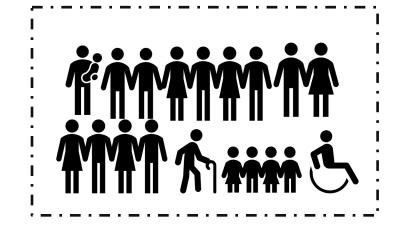
# Inferential Statistics and Significance Tests

## Populations vs Samples



A parameter is a characteristic of a population.

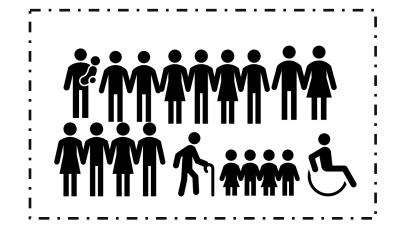
A **statistic** is a characteristic of a **sample**.





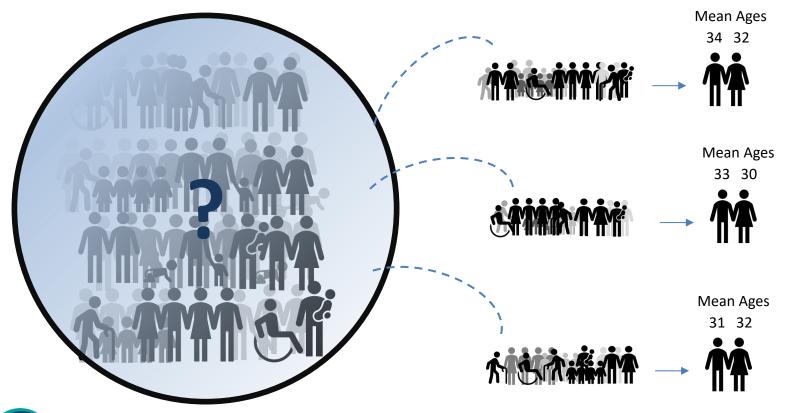
But how can we <u>infer</u> the parameter value of a population from a sample statistic?







#### Especially when different samples from the same population produce different statistics?





## Statistics is the science of variation

#### What's the likelihood that the mean ages are actually the same in the population?





## So where does the 'significance' come from?

- 'Significant' does not mean *noteworthy* instead these 'tests' simply might indicate...
- That the <u>evidence does not support</u> the assumption there's <u>no difference</u> between certain groups in the population
- This is usually measured by a probability value:

- Sig: 0.001 0.1%

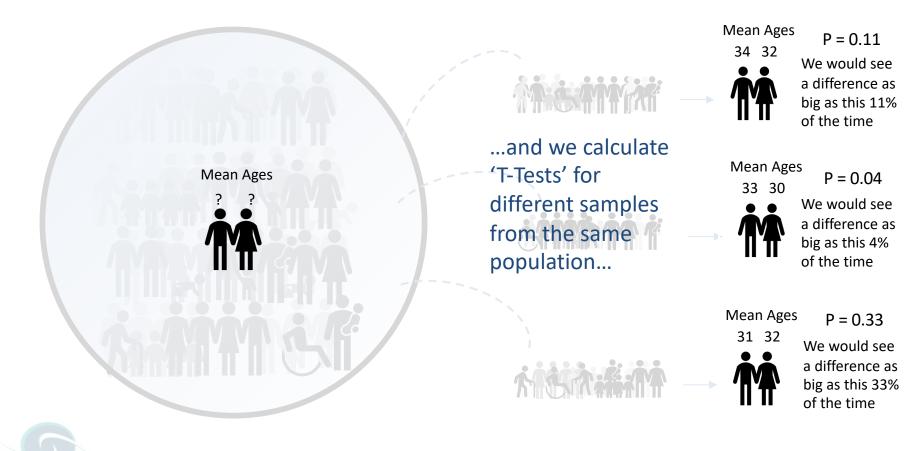
- P = 0.172 17.2%

Prob = 0.000Too small to display

Asymptotic Significance: 0.034



## If we assume there is no difference (or no relationship) between the groups...



## Significance Testing = Hypothesis Testing

- If you are investigating a relationship with statistics, it's usually because you are curious to see if there is a difference or pattern
- In other words....you have a hypothesis....
- E.g. I think that there is a difference between the average age of men and women in my population this is called the *Alternative Hypothesis* (or 'H1' for short)
- But in reality, you are testing to see if your data supports the idea that there is *no difference* between the average ages of men and women in the population of interest this is called the *Null Hypothesis* (or 'HO' for short)
- The relevant test then calculates a probability that indicates:
  - 1. If the null hypothesis is true...
  - 2. How often would we get a result as extreme as the one we observe?



## Accepting or Rejecting the Null Hypothesis

- How small (or how large) does this probability value need to be before we decide that the data doesn't provide enough evidence to support the null hypothesis?
- P= 0.32 ? Does 32% seem to be a reasonable threshold?
- What about 100%?
- Clearly, we have to choose some threshold...
- In fact, threshold is known as the Alpha Level and it's fairly arbitrary
- RA Fisher suggested 5% or P= 0.05....or a 1 in 20 chance
- So if the Probability is P=0.05 or greater, the convention is to accept the null hypothesis
- Conversely if the Probability is less than P=0.05 ....we *reject* the null hypothesis



## Interpreting 'P' values

- Contrary to a lot of statistical teaching, the 'P' value does not indicate:
  - The probability that the null hypothesis is true
  - The probability that the data were produced by random chance
- What P values can do, is indicate:
  - How compatible/incompatible the data are with a null hypothesis



## This might sound like semantics but it's not...

- For a start, the probability of x given y is not the same as the probability of y given x
  - E.g. What's the probability of someone having a driving license given they are aged 17 or over?
- Versus:
  - What's the probability of someone being aged 17 or over given that they have a driving license?
- Therefore:
  - The probability of null hypothesis being true given the evidence
- Is *not* the same as:
  - The probability of getting that evidence assuming the null hypothesis is true



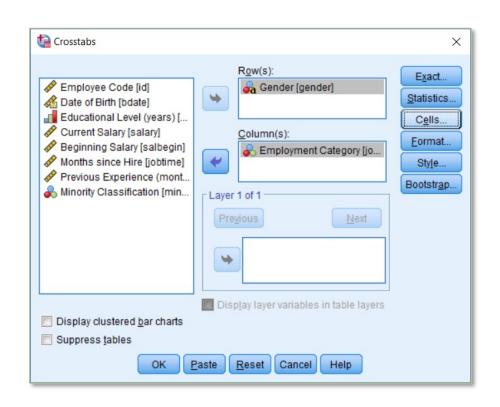
# The null hypothesis is pretty important in these 'tests of significance'

# To correctly interpret any 'significance test' you must know the Null Hypothesis associated with it

- The Null Hypothesis for a Chi Square test is that the two variables are independent of one another in the population (i.e. they are not related)
- The Null Hypothesis for a T test or an F Test is that the group *means are the same* in the population (i.e. they are the same value)
- The Null Hypothesis for a Pearson's Correlation is that the correlation value is actually zero in the population (i.e. no linear relationship)
- The Null Hypothesis for Levene's Test of Equality of Variance is that the groups have the *same spread* (or standard deviation values) in the population
- The Null Hypothesis for KS-Lilliefors test is that the variable is normally distributed in the population

## Crosstabs and Chi Square Tests

- Crosstabs are a powerful way to examine relationships between categories
- The Chi-Square test is commonly used with crosstabs as an associated statistical test





- Crosstabs are a powerful way to examine relationships between categories
- Crosstabs normally display actual frequency counts
- But they are hard to interpret if the group sizes are different

#### Gender \* Employment Category Crosstabulation

#### Count

		Emp	Employment Category			
		Clerical	Custodial	Manager	Total	
Gender	Female	206	0	10	216	
	Male	157	27	74	258	
Total		363	27	84	474	



- So they are often shown with row and/or column percentages
- In this example we have row percentages so we can compare gender in terms of employment category

Gender \* Employment Category Crosstabulation

			Emp			
			Clerical	Custodial	Manager	Total
Gender	Female	Count	206	0	10	216
		% within Gender	95.4%	0.0%	4.6%	100.0%
	Male	Count	157	27	74	258
		% within Gender	60.9%	10.5%	28.7%	100.0%
Total		Count	363	27	84	474
		% within Gender	76.6%	5.7%	17.7%	100.0%



 But Crosstabs can also display expected counts

$$\frac{216 \times 363}{474} = 165.4$$

#### Gender \* Employment Category Crosstabulation

			Emp	Employment Category			
			Clerical	Custodial	Manager	Total	
Gender	Female	Count	206	0	10	216	
		Expected Count	165.4	12.3	38.3	216.0	
	Male	Count	157	27	74	258	
		Expected Count	197.6	14.7	45.7	258.0	
Total		Count	363	27	84	474	
		Expected Count	363.0	27.0	84.0	474.0	



The differences
 between the observed
 and expected counts
 are called the residuals

**Gender \* Employment Category Crosstabulation** 

			Emp	Employment Category			
			Clerical	Custodial	Manager	Total	
Gender	Female	Count	206	0	10	216	
		Expected Count	165.4	12.3	38.3	216.0	
		Residual	40.6	-12.3	-28.3		
	Male	Count	157	27	74	258	
		Expected Count	197.6	14.7	45.7	258.0	
		Residual	-40.6	12.3	28.3		
Total		Count	363	27	84	474	
		Expected Count	363.0	27.0	84.0	474.0	



- Performing a calculation based on the sum of the squared residuals, allows us to reach a value where the larger the number, the less likely it is that the variables are unrelated to each other in the population.
- This value is the Pearson Chi-Square statistic which in turn allows us to calculate a probability value

Gender \* Employment Category Crosstabulation

			Emp	loyment Cate	gory	
			Clerical	Custodial	Manager	Total
Gender	Female	Count	206	0	10	216
		Expected Count	165.4	12.3	38.3	216.0
		% within Employment Category	56.7%	0.0%	11.9%	45.6%
	Male	Count	157	27	74	258
		Expected Count	197.6	14.7	45.7	258.0
		% within Employment Category	43.3%	100.0%	88.1%	54.4%
Total		Count	363	27	84	474
		Expected Count	363.0	27.0	84.0	474.0
		% within Employment Category	100.0%	100.0%	100.0%	100.0%

#### Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)
Pearson Chi-Square	79.277 <sup>a</sup>	2	.000
Likelihood Ratio	95.463	2	.000
N of Valid Cases	474		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 12.30.



- The null hypothesis associated with Pearson Chi-square test is that the variables are unrelated
- A small probability value (less than 0.05) indicates that differences as large as the ones observed, will only occur quite rarely if we assume this null hypothesis is true
- We therefore reject the null hypothesis

Gender \* Employment Category Crosstabulation

			Emp	Employment Category			
			Clerical	Custodial	Manager	Total	
Gender	Female	Count	206	0	10	216	
		Expected Count	165.4	12.3	38.3	216.0	
		% within Gender	95.4%	0.0%	4.6%	100.0%	
	Male	Count	157	27	74	258	
		Expected Count	197.6	14.7	45.7	258.0	
		% within Gender	60.9%	10.5%	28.7%	100.0%	
Total		Count	363	27	84	474	
		Expected Count	363.0	27.0	84.0	474.0	
		% within Gender	76.6%	5.7%	17.7%	100.0%	

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Ī	Likelihood Ratio	95.463	2	.000
	N of Valid Cases	474		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 12.30.



- Reporting a chi-square result:
- "The Chi Square test indicates that the probability of obtaining a value as extreme as the one observed, is less than 0.01 assuming the variables gender and employment category are unrelated."
- "Given this result, there appears to be insufficient evidence to support the null hypothesis and it is therefore rejected."

#### Gender \* Employment Category Crosstabulation

			Emp	Employment Category			
			Clerical	Custodial	Manager	Total	
Gender	Female	Count	206	0	10	216	
		Expected Count	165.4	12.3	38.3	216.0	
	<u> </u>	% within Gender	95.4%	0.0%	4.6%	100.0%	
	Male	Count	157	27	74	258	
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#### Chi-Square Tests

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Likelihood Ratio	95.463	2	.000
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a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 12.30.

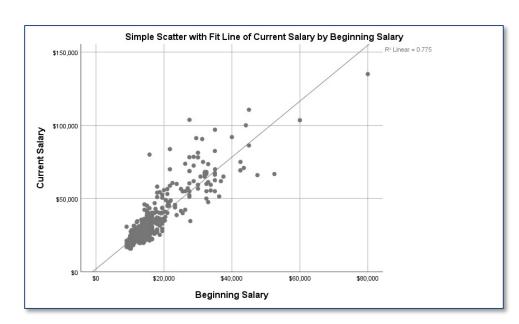


## **Correlation Coefficients**

(and when is significant not that significant)

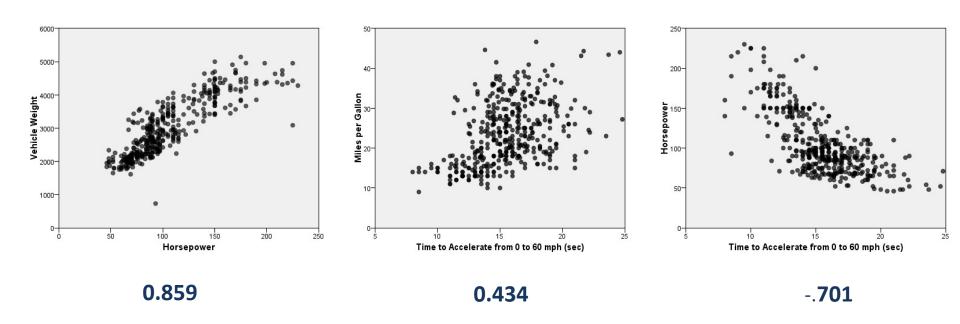
## Scatterplots to are used to illustrate relationships between continuous variables

- Here, the scatterplot shows a strong linear relationship between salary and beginning salary
- Correlation values allow us to summarise these relationships





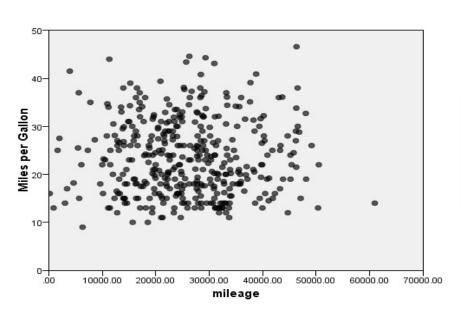
## Correlations measure the strength of *linear* relationships

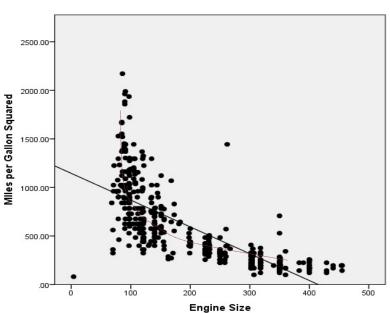


## Pearson Correlation Values\*



## Non-linear relationships are not accounted for





-0.005 -.671



## **Correlation Matrix**

• Here are the correlation values for each pair of relationships between four variables...

#### Correlations

#### Pearson Correlation

	Current Salary	Beginning Salary	Months since Hire	Previous Experience (months)
Beginning Salary	.880			
Months since Hire	.084	022		
Previous Experience (months)	097	.046	.002	
Educational Level (years)	.661	.633	.047	252



• But which ones are 'statistically significant'?

#### Correlations

#### Pearson Correlation

	Current Salary	Beginning Salary	Months since Hire	Previous Experience (months)
Beginning Salary	.880			
Months since Hire	.084	022		
Previous Experience (months)	097	.046	.002	
Educational Level (years)	.661	.633	.047	252



#### Remember this?

- The Null Hypothesis for a Chi Square test is that the two variables are independent of one another in the population (i.e. they are not related)
- The Null Hypothesis for a T test or an F Test is that the group means are the same in the population (i.e. they are the same value)
- The Null Hypothesis for a Pearson's Correlation is that the correlation value is actually zero in the population (i.e. no linear relationship)
- The Null Hypothesis for Levene's Test of Equality of Variance is that the groups have the same spread (or standard deviation values) in the population
- The Null Hypothesis for KS-Lilliefors test is that the variable is normally distributed in the population

• The null hypothesis is that the actual correlations in the population are zero i.e. there is no relationship between the pairs of variables

#### Correlations

#### Pearson Correlation

	Current Salary	Beginning Salary	Months since Hire	Previous Experience (months)
Beginning Salary	.880			
Months since Hire	.084	022		
Previous Experience (months)	097	.046	.002	
Educational Level (years)	.661	.633	.047	252

Let's add the 'significance' values



- Look at the highlighted cell. The correlation is -0.097 i.e. extremely weak
- But the significance value is 0.034 (3.4%) i.e. below P = 0.05

#### Correlations

		Current Salary	Beginning Salary	Months since Hire	Previous Experience (months)
Beginning Salary	Pearson Correlation	.880			
	Sig. (2-tailed)	<.001			
	N	474			
Months since Hire	Pearson Correlation	.084	022		
	Sig. (2-tailed)	.067	.626		
	N	474	475		
Previous Experience	Pearson Correlation	097	.046	.002	
(months)	Sig. (2-tailed)	.034	.319	.974	
	N	474	475	475	
Educational Level (years)	Pearson Correlation	.661	.633	.047	252
	Sig. (2-tailed)	<.001	<.001	.310	<.001
	N	474	475	475	475



• Is this statistically significant? Yes – because the probability of getting a result as 'extreme' as -0.097, assuming there is no relationship between the two variables in the population is still only 0.034 (3.4%)

Correlations

		Current Salary	Beginning Salary	Months since Hire	Previous Experience (months)
Beginning Salary	Pearson Correlation	.880			
	Sig. (2-tailed)	<.001			
	N	474			
Months since Hire	Pearson Correlation	.084	022		
	Sig. (2-tailed)	.067	.626		
	N	474	475		
Previous Experience	Pearson Correlation	097	.046	.002	
(months)	Sig. (2-tailed)	.034	.319	.974	
	N	474	475	475	
Educational Level (years)	Pearson Correlation	.661	.633	.047	252
	Sig. (2-tailed)	<.001	<.001	.310	<.001
	N	474	475	475	475



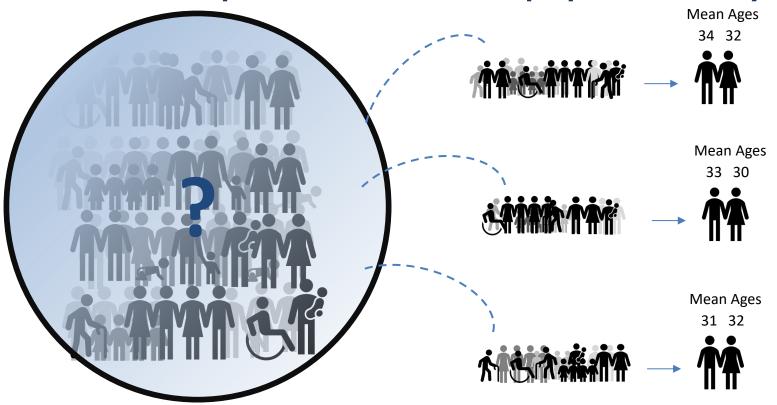
• Does that mean it is notable or worth reporting? No, not particularly. But then that's not what the null hypothesis for a Pearson's correlation value is directed at. It only states that there is no relationship there at all in the population.

		Current Salary	Beginning Salary	Months since Hire	Previous Experience (months)
Beginning Salary	Pearson Correlation	.880			
	Sig. (2-tailed)	<.001			
	N	474			
Months since Hire	Pearson Correlation	.084	022		
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Previous Experience	Pearson Correlation	097	.046	.002	
(months)	Sig. (2-tailed)	.034	.319	.974	
	N	474	475	475	
Educational Level (years)	Pearson Correlation	.661	.633	.047	252
	Sig. (2-tailed)	<.001	<.001	.310	<.001
	N	474	475	475	475



# How to interpret confidence intervals correctly

## Different samples from the same population vary





## Different samples give different results

- Repeat a survey or a project and you won't get exactly the same results calculate a statistic and the results vary from one sample to the next.
- No statistical calculation can tell you what the actual value of a population parameter is and we don't have the luxury of repeating samples over and over again
- But we can estimate <u>a</u> range of values that it is likely to lie within
- We can do this by requesting *Confidence Intervals*
- Confidence Intervals can be shown graphically as Error Bars





- The 'Explore' procedure in SPSS produces lots of summary measures...
- Here it says the mean age for employees in this data sample was 34.74
- But it also provides 95%
  Confidence Intervals

#### Descriptives

			Statistic	Std. Error
Age of Employee	Mean		34.74	.542
	95% Confidence Interval	Lower Bound	33.67	
	for Mean	Upper Bound	35.80	
	5% Trimmed Mean		34.06	
	Median		29.00	
	Variance		139.034	
	Std. Deviation		11.791	
	Minimum		20	
	Maximum		62	
	Range		42	
	Interquartile Range		18	
	Skewness		.859	.112
	Kurtosis		573	.224



- So in this example, the mean age *happens* to be **34.74**
- Of course, we don't know what the mean age for employees is in the population that the sample was drawn from, but the confidence intervals indicate that it is likely to be contained in a range like 33.67 to 35.8
- As we know, drawing another sample would probably result in a slightly different mean value
- But <u>all our statistics</u> are likely to vary from one sample to the next not just the mean



- So, if we drew another comparable sample and recalculated the confidence intervals, they too would probably be different
- In other words, we can't get too hung-up on the values of the confidence intervals themselves, any more than we can fixate on the precise value of the mean
- What we can say however, is that even if the intervals vary from one sample to the next, they are likely to be broad enough that on 95% of occasions, they will contain the population mean (i.e. the parameter)



- What we should not say is:
- "We are 95% confident that the population parameter is between X and X"
- Because:
  - The intervals themselves will vary from sample to sample
  - There's no scientific basis to the phrase 'we are 95% confident'
  - It is the **procedure** that on 95% of occasions will capture the population mean



#### How are confidence intervals calculated?

- The special statistic that drives confidence intervals is a called a standard error
- When calculating confidence intervals for a mean value, we use the standard error of the mean
- But there are other standard error statistics such as the standard error of the median, the standard error of the difference, the standard error of the correlation etc.
- These are useful for when we need to calculate confidence intervals for other statistics



#### How are confidence intervals calculated?

- A standard error is based on a standard deviation
- Indeed just as a standard deviation measures variation *within* a sample, the standard error measures variation *between* samples
- So the standard error of a mean tells us on average how much we would expect a sample mean to vary from one sample to another

De			

			Statistic	Std. Error
Age of Employee	Mean		34.74	.542
	95% Confidence Interval	Lower Bound	33.67	
	for Mean	Upper Bound	35.80	
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	Range		42	
	Interquartile Range		18	
	Skewness		.859	.112
	Kurtosis		573	.224



#### How are confidence intervals calculated?

- To calculate 95% confidence intervals...
- We multiply a standard error by roughly two (or 1.96 to be exact)
  - 0.542 x 1.96 = 1.062
- We then add and substract this value from the mean to get our Confidence Intervals
- 34.74 1.062 = **33.678**
- 34.74 + 1.062 = **35.802**

			Statistic	Std. Error
Age of Employee	Mean		34.74	.542
	95% Confidence Interval	Lower Bound	33.67	
	for Mean	Upper Bound	35.80	1
	5% Trimmed Mean		34.06	
	Median		29.00	
	Variance		139.034	
	Std. Deviation		11.791	
	Minimum		20	
	Maximum		62	
	Range		42	
	Interquartile Range		18	
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Descriptives



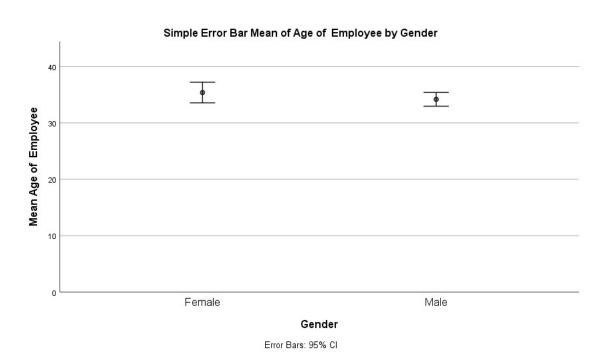
#### Wide intervals vs narrow intervals

- Because the standard error is based on the standard deviation and
  The sample size...
- The width of the confidence intervals is affected by two values:
  - 1. The size of the sample
  - 2. The spread in data (the standard deviation of the variable)
- So confidence intervals move further apart:
  - 1. The smaller the sample
  - 2. The greater the spread (i.e. the larger the standard deviation)
- And confidence intervals move closer together:
  - 1. The larger the sample
  - 2. The lesser the spread (i.e. the smaller the standard deviation)



## Graphing Confidence Intervals with Error Bars

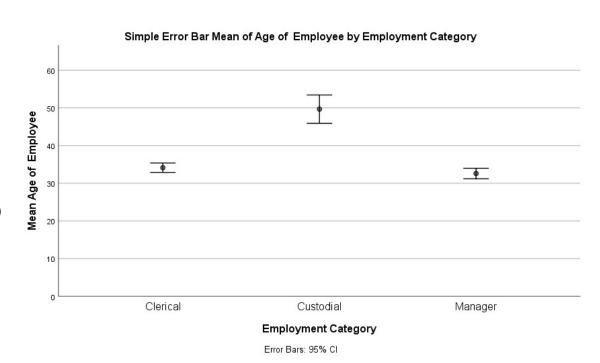
- The dot in the middle represents the mean while the upper and lower bars represent the upper and lower confidence intervals
- Note that the bars don't quite cover the same range of values
- Note that the error bars overlap





## Graphing Confidence Intervals with Error Bars

- Contrast the previous example with the variable Employment Category
- Note that in the Custodial categories the CI intervals do not overlap with the other two groups



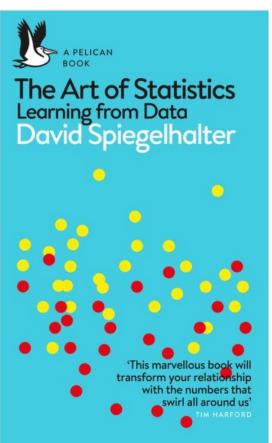


## Further Reading: the 'Eat Your Greens' blog series

- Just because something is statistically significant doesn't mean it's practically significant
- 2. Testing versus inferring
- 3. What's 'standard' about a standard deviation?
- 4. <u>Finding normality why is the normal distribution so important when we so rarely encounter it in real life?</u>
- 5. The gateway to inference the standard error and confidence intervals
- 6. <u>Understanding correlation</u>
- 7. Making sense of significance tests
- 8. <u>Introduction to Power Analysis</u>



#### Further Reading: Recommended

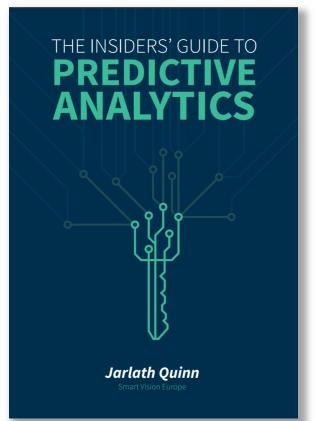


The Art of Statistics Learning from Data - Pelican Books

D. J. Spiegelhalter (author)



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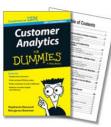
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#### Training and Consulting Services

- Guided consulting & training to develop in house skills
- Delivery of classroom training courses / side by side training support
- Identification & recruitment of analytical skills into your organisation

#### Advice and Support

- offer 'no strings attached' technical and business advice relating to analytical activities
- Technical support services



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## Thank you